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On Transversal and Y-Extremal Hypersurfaces

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ABSTRACT -
We consider torsion free recurrent-connection, The induced linear Y-connection recurrency with respect to h-recurrent Finsler Connection transversal hyper-surface.

I. INTRODUCTION :-

Theory of hyper-surfaces in Finsler space has been first considered by E.Cartan [1] from two point of view. One is to regard a hyper-surface as the whole of tangent line elements then it is also a Finsler Space [3]. The other is to regard it as whole of normal line elements then it is a Riemannian Space. J.M. Wagner ([4]) has treated hyper-surfaces from latter view point and dealt in particular with minimal hyper-surfaces.

II. TRANSVERSAL HYPERSURFACE :-

We are concerned with torsion free recurrent Finsler Connection Rec. Ɩ(a) for which $T^i_{jk} = 0$ then (2.1) is -

(2.1) $\bar{g}_{ir}\,(Y^r_j - Y^r_k\,Y_j\quad Y^k) + \frac{1}{2} Y_i\quad \bar{a}_j = \bar{g}_{jr}\,(Y^r_i - Y^r_k\,Y^k\,Y_i\quad) + \frac{1}{2} Y_j\quad \bar{a}_i$

In this (2.2) case fs Y is auto parallel, then (2.2) reduces to

(2.2) $\bar{g}_{ir}\,Y^r_j + Y_i\,(T_{jrk}\,Y^r\,Y^k + \frac{1}{2}\,\bar{a}_j) - i/j\,\} + Yr\,T^r_{ij} = 0$

(2.2) $Yg^{ij}\ Y^r + \frac{1}{2}\ Y_i\ \bar{a}_j = \bar{g}_{jr}\ Y^r_i + \frac{1}{2}\ Y_j\ \bar{a}_j$

Now we consider the geometry of transversal hyper-surface M^{n-1} : $x^i = x^i\ (\mu^\alpha)$. The vector field Y normalized by Finsler metric L i.e.

(2.3) $g_{ij}\,(x\ y)\ Y^i\ Y^j = 1$ the unit vector field or thogonal to M^{n-1} in sense of

(2.4) gij (x,y) $Y^i\ B^j_\alpha\ = 0\quad \alpha$=1------n-1 So we get a field of frame

($B^i_\alpha Y^i$) along M^{n-1} further from.

(2.5) $\bar{g}_{ir}\,(x) = g_{ij}\,(x, y)$

We get induced Riemannian Y metric on M^{n-1}

(2.6) $g\alpha\beta = g_{ij}\,(x, y)\ B^i_\alpha\ B^j_\beta$

and have a linear connection Ɩ(Y) induced from the linear Y-connection Y (┌).

Denoting (B^α_i, Y^i) the dual cofrome of (B^i_α, Y^i) The connection coefficient $┌^2_{\beta\gamma}$ (u) of Ɩ(Y) are

$$┌^\alpha_{\beta\gamma} = B^\alpha_i \qquad (B^i_{\beta\gamma} + B^i_\beta\ \overline{\sqrt{jk}}^{\,i} \qquad B^k_\gamma)$$

$$B^i_\beta = \frac{\partial B^i_\beta}{\partial u^r}$$

So we get so called gauss equ.

(2.7) $B^i_\alpha;\beta = H_\alpha\beta\ Y^i$

Where denote the relative covariant differentiation along M^{n-1} i.e.

(2.8) $B^i_\alpha;\beta = B^i_{\alpha\beta} + B^r_\alpha\ \overline{\sqrt{jk}}^{\,i}\quad B^k_\beta - P^i\ ┌^r_{\alpha\beta}$

and $H_\alpha\beta$ is the second fundamental tensor of M^{n-1}

So we obtain. So called Weingarte eq.

(2.9) $Y^i_{;\beta} = H^\alpha_\beta\ B^i_\alpha - \frac{1}{2}\ a_\beta\quad Y^i$

It is obvious that $\sqrt{(y)}$ is also recurrent with respect to induced Riemannian Y-metric

$g_\alpha\beta;_\gamma = a\gamma\ g_\alpha\beta$

From (2.10) and (2.11)

$B^i_{\alpha;\beta} = H_{\alpha\beta}\ Y^i$

(2.11) $B^i_{\alpha;\beta} = B^i_{\alpha\beta} + B^r_\alpha\ \overline{\sqrt{jk}}^{\,i}\quad B^k_\beta - B^i_\gamma\ \overline{\sqrt{\alpha\beta}}^{\,\gamma}$

torsion tensor $T^\alpha_{\beta\gamma}$ of $\sqrt{(y)}$ and $H_{\alpha\beta}$ $H_{\beta\alpha}$ are given by

(2.12) $T^\alpha_{\beta\gamma} = B^\alpha_i\ \bar{T}^i_{jk}\ B^j_\alpha\ B^i_l$

(2.13) $H_{\alpha\beta} - H_{\beta\alpha} = Y_i\ \bar{T}^i_{jk}\ B^j_\alpha\ B^k_\beta$

We consider the Y_2 tensor field Y^i_j along M^{n-1} . The relative covariant derivative $Y^i_{;\beta}$of Y^i is defined as

$$Y^i_{;\beta} = \frac{\partial Y^i}{\partial u^\beta} + Y^i\ \overline{\sqrt{jk}}^{\,i}\quad B^k_\beta - Y^i_k\ B^k_\beta$$

So if we write Y^i_j with respect to frame (B^i_α Y^i) we have

$Y^i_j = B^\alpha_\beta\ B^i_\alpha\ B^\beta_j - \frac{1}{2}\,a_\beta\,B^\beta_j\ Y^i\,(Y^\alpha\,B^i_\alpha + YY^i)\ Y_j$

for some function Y^α and Y. Further more

(2.14) $Y_r\ Y^r_i = -\frac{1}{2}\ \bar{a}_k$

So we have

(2.15) $Y_j^i = B_\alpha^i$ (- $B_\beta^\alpha B_j^\alpha$ + Y Y_j^α) - $\frac{1}{2}$ a_j Y^i

In Particular we shall deal with torsion free recurrent Finsler connection Rec $\sqrt{(a)}$ (T=0) then

(2.16) $\bar{T}_{jk}^i$(x) = T_{jk}^i (x, y) + C_{jr}^i (x, y) Y_j^r - C_{kr}^i (x, y) Y_j^r

reduce ɔ

$\dot{T}_{jk}^i = C_{jr}^i$ (x, y) Y_k^r - C_{kr}^i (x, y) Y_j^r

From (2.12) we get

(2.17) $T_\alpha\beta\gamma = H_{\alpha\rho}\ C_{\beta\gamma}^\rho - H_{\gamma\rho}\ C_{\beta 2}^\rho$

Where

$C_{\beta\gamma}^\alpha = C_{jk}^i$ (x,y) $B_i^\alpha B_\beta^j B_\gamma^k$

Proposition 2.1 :-

In case of torsion free recurrent Finsler Connection Rec. $\sqrt{(a)}$ the induced lineary connection $\sqrt{(y)}$ is recurrent with recurrence vector α_β with respect to induced Riemarⁿian Y-metric and torsion tensor of $\sqrt{(y)}$ is ᶇ ᵞen by (2.17)

III. Y-EXTREMAL HYPERSURFACE :-

A transversal hyper-surface M^{n-1} (c) is called a Y-external hyper-surface if it satisfied equation.

3.1) { $\frac{\partial\sqrt{g}}{\partial x_i^i}$ - ∂($\frac{\partial\sqrt{g}}{\partial B_i^\alpha}$)/ ∂u^2} Y^i = 0

Differentiation of

(3.2) $g_{\alpha\beta}$ (u) = gij (x,y) B_α^i B_β^j gives

$\frac{\partial g}{\partial B_\alpha^i}$ = ($\frac{\partial g\beta\gamma}{\partial B_\alpha^\gamma}$) g $g\beta\gamma$ = gjk (x,y)

{ $\frac{\partial (B_\beta^r\ B_\gamma^k}{\partial B_{\alpha 2}^i}$} $g g^{\beta\gamma}$ = 2g B_i^α

So we have

(3.3) $\frac{\partial\sqrt{g}\,B^\alpha}{\partial B_\alpha^i}$ = $\sqrt{g}\ B_i^\alpha$

We shall rewrite equation (3.1) from (3.2) and recurrent properties of $\sqrt{(y)}$ we have

$\frac{\partial\sqrt{g}}{\partial x^i}$ = $\frac{\sqrt{g}}{2}$ ($\frac{\partial g\alpha\beta}{x^i}$) $g^{\alpha\beta}$

= $\frac{\sqrt{g}}{2}$ ($\frac{\partial g 2\beta}{\partial x^i}$) B_i^α B_β^k $g^{\alpha\beta}$

= ($\frac{n-1}{2}$) $\sqrt{g}\ \bar{a}_i$+ $\sqrt{g}$ ($\bar{g}^{ik}$ - Y^j Y^k) $\sqrt{jki}$

(3.4) $\frac{\partial\sqrt{g}}{\partial x^i}$ = ($\frac{n-1}{2}$) $\sqrt{g}\ \bar{a}\imath$ + $\sqrt{g}$ ($\sqrt{\quad r\imath}$ - $\sqrt{rs\imath}$ Y^r Y^s)

Next from (3.3) we have

(3.5) $\frac{\partial}{\partial u^\alpha}$ ($\frac{\partial\sqrt{g}}{\partial B_\alpha^i}$) = $\frac{\partial\sqrt{g}}{\partial u^\alpha}$ B_i^α + $\sqrt{g}$ $\frac{\partial B_i^\alpha}{\partial u^\alpha}$

Again

(3.6) $B_{\alpha;\beta}^i = H_{\alpha\beta}$ Y^i

and (3.7) $Y_{;\beta}^i = Y_\beta^\alpha\ B_\alpha^i$ - $\frac{1}{2}$ a_β Y^i

(3.7) $\frac{\partial B_i^\alpha}{\partial u^\alpha}$ B_α^i Y_i - B_i^Y $\sqrt{\gamma_\alpha^\alpha}$ $\frac{\alpha}{2}$ + (δ_j^k- Yj Yl $\sqrt{\overline{\quad}_{\imath k}^{J}}$

So we obtain

(3.8) ∂ ($\frac{\partial\sqrt{g}}{\partial B_\alpha^i}$) / ∂u^2 = $\sqrt{g}$ { $\frac{(n-1)}{2}$ a_α B_i^α + H_α^α Y_i + $T\rho_\alpha^o$ B_i^α + $\overline{\Gamma_{\imath r}^r}$ - $\overline{\Gamma_{\imath rs}}$ Y^r Y^s }

as consequence of (3.4) (3.8) term in curly bracket of eq. (3.1) becomes.

(3.9) $\sqrt{g}$ {$\overline{T_{r\imath}^r}$ + $\bar{T}_{irs}$ Y^r Y^s - H_α^α Y_i - $T_{\rho\alpha}^\rho$ B_i^α + $\frac{n-1}{2}$ a_r Y^r Y_i }

from eq.

(4.0) $\bar{T}_{jk}^i$ (x) = T_{jk}^i (x, y) + C_{jk}^i (x, y) Y_k^r - C_{kr}^i (x, y) Y_j^r

(3.1) is written in the term

(4.1) M = [T_{jk}^i (x, y) + C_j (x, y) Y_k^j + $\frac{n-1}{2}$ B_α^i] Y^k

Where M = $g^{\alpha\beta}$ H_α β= H_α^α is called mɛ ı curvature.

Theorem :-

(3.1) - With respect to h-recurrent Finsler Connection Rec $\sqrt{(T,a)}$ a transversal hyper-surface is Y-extermal is eq. (4.1) is satisfied.

REFERENCES

[1]. Barlhel W.: Uber die Minimal Sevchen in gesaserten Finsler Umen Ann. 41 Mat, (4), 36 (1954) 159-190.

[2]. Cartan E.: Lee espaces de Finsler Actualities 79 Paris (1934).

[3]. Malsumoto M.: The induced and intrinsic Finsler Connection of hyper-surface and Finslerian Projective geometry.

[4]. Wagner J.M.: Untereuchungen Uber Finsler Chen R Uame Lotos Prag, 84 (1ᵔ⁻6).